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Dynamic Analysis of the Flat Spin Mode of a General Aviation Aircraft

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The properties of the flat spin mode of a general aviation configuration have been studied through analysis of rotary balance data. The equilibrium state is predicted well from rotary balance data. Linearized analyses about the flat spin attitude show the existence of two-coupled pitch-roll modes and a decoupled yaw mode. The stability of the flat spin mode has been examined extensively using numerical linearization, classical perturbation methods, and reduced order models. The stability exhibited by the time histories and the eigenvalue analyses is shown to be strongly dependent on the oscillatory aerodynamic derivatives. Explicit stability criteria are obtained from the reduced order models.

Nomenclature

b	= wing span, ft
c.g.	= center of gravity, percent mean aerodynamic chord
I_x, I_y, I_z	= moments of inertia about the x, y, and z axes, respectively, slug-ft^2
L, M, N	= rolling, pitching, and yawing moments, respectively, ft-lb
L_I	= gyroscopic rolling moment $= (I_y - I_z)\, qr$, ft-lb
L_p	= rate of change of rolling moment with roll rate $= (1/I_x)(\partial L/\partial p)$, s^{-1}
L_{q_I}	= rate of change of gyroscopic rolling moment with pitch rate $= (1/I_x)(\partial L_I/\partial q)$, s^{-1}
L_r	= rate of change of rolling moment with yaw rate $= (1/I_x)(\partial L/\partial r)$, s^{-1}
L_ϕ	= rate of change of rolling moment with roll angle $= (1/I_x)(\partial L/\partial \phi)$; for constrained flat spins, $L_\phi \doteq L_\beta$, s^{-2}
m.a.c.	= mean aerodynamic chord, ft
M_I	= gyroscopic pitching moment $= (I_z - I_x)\, rp$, ft-lb
M_p	= rate of change of pitching moment with respect to roll rate $= (1/I_y)(\partial M/\partial p)$, s^{-1}
M_{p_I}	= rate of change of gyroscopic pitching moment with roll rate $= (1/I_y)(\partial M_I/\partial p)$, s^{-1}
M_q	= rate of change of pitching moment with pitch rate $= (1/I_y)(\partial M/\partial q)$, s^{-1}
M_α	= rate of change of pitching moment with angle of attack $= (1/I_y)(\partial M/\partial \alpha)$, s^{-2}
M_β	= rate of change of pitching moment with sideslip $= (1/I_y)(\partial M/\partial \beta)$, s^{-2}
M_ϕ	= rate of change of pitching moment with roll angle $= (1/I_y)(\partial M/\partial \phi)$; for constrained flat spins, $M_\phi \doteq M_\beta$, s^{-1}
N_p	= rate of change of yawing moment with roll rate $= (1/I_z)(\partial N/\partial p)$, s^{-1}
N_r	= rate of change of yawing moment with yaw rate $= (1/I_z)(\partial N/\partial p)$, s^{-1}
p, q, r	= roll, pitch, and yaw rate, respectively, rad/s (deg/s)
R	= steady spin radius, measured from spin axis to aircraft c.g., ft
S	= wing planform area, ft^2
V	= aircraft total velocity, ft/s
$V_{z'}$	= vertical descent speed, ft/s; for constrained helical motion $V_{z'} \doteq V_z$
x, y, z	= body axes system with origin at the c.g.; x positive forward, y positive out right wing, z positive down
α	= angle of attack at c.g., $\alpha \doteq \alpha'$, rad (deg)
α'	= angle between the x body axis and vertical (Fig. 1), positive for x axis below horizon (erect spin)
β	= angle of sideslip at c.g., positive when relative wind comes from right of plane of symmetry, rad (deg); for constrained helical motion, $\beta_{rad} \doteq -\chi_{rad} \cos\alpha' + \phi_{rad} \sin\alpha' - \gamma_{rad}$
γ	= spin helix angle $= \tan^{-1}(\Omega R/V)$, rad (deg)
ζ	= damping ratio
θ	= pitch angle between x axis and horizontal measured in the vertical plane, positive nose-up, rad (deg)
λ	= spin parameter $= \Omega b/2V$
ρ	= air density, slugs/ft^3
σ	= real part of a complex root
ϕ	= Euler roll angle between y axis and horizontal measured about the x-body axis, positive when right wing is down, rad (deg)
χ	= wing tilt angle measured about the z-body axis, positive for a nose clockwise rotation, rad (deg)
ω	= imaginary part of a complex root
Ω	= aircraft angular velocity about the vertical spin axis, positive for a right spin, rad/s (deg/s)

Subscripts

osc	= oscillatory value
rad	= angle in radians
RB	= rotary balance value
ss	= quasi-steady-state value
trim	= steady spin value

Superscript

$(\hat{\ })$	= normalized eigenvector

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Introduction

THE prediction and analysis of aircraft spin characteristics has long been of interest to the aviation community. Pioneering research in spinning motions, as published around 1930 by the British scientists Jones[1] and Bryant,[2] suffered from inaccuracies and incompleteness of their aerodynamic models, and the inability of the linearized theory to predict the highly nonlinear sideslip effects. The motion of the aircraft was analyzed using classical perturbation methods. Aerodynamic data bases were at first obtained almost entirely from static balance measurements, but later incorporated rotary balance data. (A rotary balance measures the forces and moments on a model under steady rotating conditions about an adjustable spin axis.)

Scher[3] and Anglin,[4,5] in the 1950s and 1960s, presented the results of simulation studies based on extensive rotary and oscillatory balance data bases. Although some data base inconsistencies were reported in these studies, time history results generally correlated well with dynamic model tests of military configurations. Recent research efforts, most notably by Williams,[6] Kroll,[7] and Graham,[8] have been focused on studying the dynamic modes of fighters in spins, and the identification of the parameters which are most important in determining their stability characteristics.

Graham's studies were based on a 3-degree-of-freedom (DOF) dynamic model which constrained the aircraft to travel a vertical flight path, but allowed arbitrary angular motion. Similar lower order modes were first introduced in the 1930s by Jones,[1] Gates,[9] and Irving,[10] and later by other investigators[11,12] to reduce the complexity of the spin problem. Several analysis techniques, using these reduced order models, have been demonstrated to yield reasonably accurate and efficient steady spin predictions which correlate well with full-scale military aircraft flight tests.[4,12,13]

Whereas past efforts were largely centered on military requirements, recent interest has grown in the development of spin analysis tools readily applicable to general aviation technology. Papers by Tischler and Barlow[14-16] have presented the formulation and implementation of the equilibrium spin technique, a graphical method for obtaining recovery and steady spin characteristics from rotary balance data. These references discussed the results of extensive analyses on the NASA low-wing general aviation aircraft[17] with a variety of tail configurations‡ and control settings. The calculated results showed close correlation to the available spin tunnel[18] and full-scale flight test data.[19]

Results of a preliminary dynamic analysis of the NASA aircraft with Tail Configuration Three were presented by Tischler.[16] Two significant features characterize the time history results of that study. First, pitch and roll oscillations were dominant for both steep and flat spins. (Steep spins are defined here as occurring in the angle-of-attack range $25 \leq \alpha \leq 40$; flat spins are defined here as occurring in the angle-of-attack range $60 \leq \alpha \leq 90$.) Second, despite conditions of increasing angular body rate oscillations, the aircraft c.g. continues to travel a helical path of virtually constant radius, rate of descent, and spin rate. The steadiness of the helical motion was especially prominent in the flat spin cases.

A 3-DOF model, based on the assumption of constrained c.g. helical motion and using numerical linearization methods, was developed to study the stability characteristics of the aircraft about its calculated equilibrium spin attitude. The results were found to accurately depict the relevant motions of unconstrained 6-DOF simulated time histories. However, the 6-DOF simulation results were found to be generally unstable, due to the inability of a pure rotary balance generated data base to accurately model the aerodynamics of stable spinning motion.

The present paper discusses approximate techniques, based on reduced degree-of-freedom representations, for the evaluation of the dynamic spin behavior of general aviation configurations. These techniques were used in an in-depth analysis of the dynamic motion of the NASA low-wing aircraft about its calculated flat spin attitude. (The steep spin, which is discussed elsewhere,[16] is not treated in the present study.) This analysis utilized extensive rotary balance sideslip data[20] and estimated oscillatory derivatives, not incorporated in the previous preliminary study.

One goal of the present study was to gain a better "physical insight" into dynamic spinning motion by determining the important parameters which control the stability of this motion. An additional objective of this study was the formulation of general guidelines for data base requirements in the digital simulation of spinning motion.

An iterative computer trim program was utilized to check the accuracy of the graphical results of Ref. 15. The dynamic modes of the aircraft's motion were studied using numerical linearization techniques and a 3-DOF model. These results are presented in the form of eigenvalue and eigenvector (Argand) diagrams.

Based on the results of the modal analyses, simplified 2- and 1-DOF models were developed in order to analytically determine stability criteria. A presentation of these criteria and a comparison of the lower order models with the earlier validated 3-DOF model are given in this paper. Unconstrained 6-DOF time histories for this configuration are also presented for comparison with full-scale flight test data,[21] and validation of the lower order linearized models.

Analysis Technique

To provide a basis for comparison among simpler schemes, a 6-DOF digital simulation program was developed to numerically analyze the dynamic motion of spinning aircraft. This program incorporated an iterative trim subroutine, which accessed the 6-DOF dynamic and aerodynamic models, to obtain an exact solution for the steady spin conditions. A generalized secant algorithm[22] was used to search for the exact trim solution, starting from the approximate (equilibrium spin technique) results of Ref. 15.

The program also included a subroutine which, through the application of numerical perturbation techniques, evaluated the characteristic system ("A") matrix of spinning aircraft at their trim condition, using the 3-DOF (helical motion) model of Ref. 16. The corresponding eigenvalues and eigenvectors were evaluated by standard computer library routines and were plotted in Argand diagram (time vector) form.

Time history analyses were initialized with the trim solution subroutine results. Excitation away from the trim attitude was generated by a perturbation in angle of attack. The ensuing motion, for a 10 s duration, was obtained by numerical integration of the body axis nonlinear equations with a fixed time step, fourth order, Runge-Kutta scheme.

The major emphasis of the present study, as discussed earlier, was to determine the important parameters which control the motion of aircraft in spins. This was accomplished by a classical perturbation analysis of reduced order model dynamic equations. Basic results from the theory of differential equations were employed to determine analytical stability criteria. Simplifications, based on order-of-magnitude considerations, were used to obtain approximate analytical expressions for the values of the characteristic equation roots (eigenvalues). A discussion and comparison of these results and the results obtained by the numerical linearization technique is now presented.

Aerodynamic Model

Rotary balance wind-tunnel data for the NASA low-wing aircraft with Tail Configuration Four and the c.g. at 25.5% of

‡Tail configurations are distinguished primarily by the vertical positioning of the horizontal stabilizer. Configuration Four has a horizontal tail location on the fuselage, somewhat below that of Configuration Three.[17]

the m.a.c. were available from Refs. 17 and 20. This data base covered a large variety of control settings and an angle-of-attack range of 30-90 deg but not sideslip. The nondimensional spin rate λ ($=\Omega b/2V$) was varied from -0.9 to $+0.9$. Sideslip data were obtained by rolling the model ± 10 deg with no control deflection. The sideslip (no control deflection) data and control deflection (no sideslip) data were combined by superposition. The present study utilized the data for settings of ailerons neutral, rudder full right (pro-spin), and elevator full up (pro-spin). The assumption of superposition was felt to provide representative results because the "neutral ailerons" case exhibits the least amount of sideslip asymmetry. The data base used in the present study did not include spin radius effects. These effects are reported to be small,[23] especially in the flat spin cases, where the full-scale radius is typically less than 1 ft, i.e., a small percentage of the wing span.

Oscillatory balance data have been shown by previous studies to provide the necessary damping in the simulation of dynamic spinning motion.

The method of combining oscillatory and rotary balance data, for use in digital simulations, is presented in detail in Refs. 4 and 7. Briefly stated, at each time step the instantaneous angular velocity vector is projected onto the resultant wind vector. This projection, Ω_{ss} (quasi-steady-state), is used along with the appropriate angles of attack and sideslip to "look up" the appropriate rotary balance measured forces and moments. The vector Ω_{ss} is then resolved onto the body axes to form p_{ss}, q_{ss}, and r_{ss}. In the simulation these quasi-steady components are subtracted from the total instantaneous angular body rates, leaving the "oscillatory components" p_{osc}, q_{osc}, r_{osc}. These rates are multiplied by the separately measured damping derivatives ($L_{p_{osc}}$, $L_{r_{osc}}$, $M_{q_{osc}}$, $N_{r_{osc}}$, etc.) (at appropriate angles of attack) to provide the desired oscillatory aerodynamic forces and moments. For instance,

$$L(\alpha,\beta,p,q,r) = \underbrace{L_{ss}(\alpha,\beta,\Omega)}_{\text{Rotary data}} + \underbrace{L_{p_{osc}}(\alpha,\beta)p_{osc} + L_{r_{osc}}(\alpha,\beta)r_{osc}}_{\text{Oscillatory data}} \quad (1)$$

These damping derivatives are normally obtained by a forced or free oscillation rig. Such data are currently scarce for unswept low-wing (general aviation) configurations. An estimate of the pitch damping, $M_{q_{osc}}$, and roll damping, $L_{p_{osc}}$, was calculated based on the fully separated, or "crossflow" technique.[24] For the present flat spin study ($\alpha \doteq 70$ deg), the assumption of full separated flow over the wing and horizontal tail was felt to be adequate. Because of the uncertainty of the flow conditions at the vertical tail (mounted completely above the horizontal tail), the yaw damping effect, $N_{r_{osc}}$, was ignored for the present study. An examination of the rotary balance data in the flat spin range indicated that the effect is probably small. The cross-coupling oscillatory derivatives (such as $L_{r_{osc}}$, $N_{p_{osc}}$, $M_{p_{osc}}$, etc.) may be important for stability analyses, but were omitted in the present study because their values were neither known experimentally nor easily estimated.

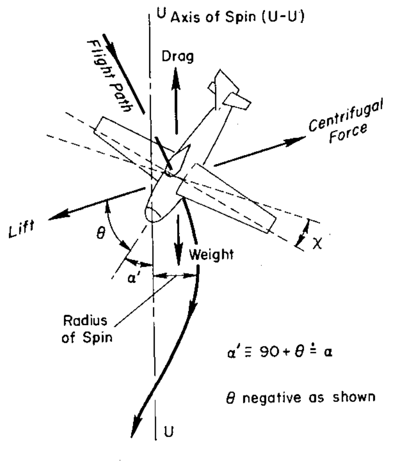

Fig. 1 Balance of forces in a steady spin.

The present study also ignored the effects of angular accelerations on the aerodynamics of the aircraft. Such effects, which have been reported to be significant in spin entry simulations, are not important for developed spin studies where angular accelerations are small.[25] This is especially true for developed flat spins.

Choice of Euler Angles

Approximate analysis techniques have historically employed the assumptions that in a spin an aircraft has its average resultant aerodynamic force vector colinear with the z-body axis and also that this vector crosses the vertical spin axis[9-12] (Fig. 1). These assumptions, which are consistent with wind-tunnel and spin-tunnel data,[17,18,20] are employed to satisfy the constant spin rate condition and to decouple the force equations.[16] In order to maintain the above geometric constraint, a nonstandard Euler angle sequence is employed to describe an aircraft's orientation in a spin.

The aircraft x and y axes are assumed to be initially in the horizontal plane with the c.g. at a distance R (spin radius) from the spin axis. The x-body axis is oriented to intersect the spin center. The aircraft is first pitched nose-down at an angle $-\theta \doteq (90-\alpha)$ and, second, yawed through an angle χ to its final steady spin orientation (Fig. 1). (A more detailed discussion of steady spin geometry is presented in Refs. 9, 10, and 16.) This Euler sequence was used in the development and subsequent application of the equilibrium spin technique.[16]

In order to model unconstrained dynamic spinning motion, the restriction on the orientation of the z-body axis must be lifted. Retaining the first two rotations to maintain consistency with the previous results, a third rotation, roll ϕ is added. This nonstandard three rotation sequence was used in all dynamic analyses for the preliminary study[16] and the present study. For an unconstrained trimmed spinning condition (steady spin), the roll angle should approach zero to be consistent with the data and assumptions of the equilibrium spin technique.

Results and Discussion

The iterative trim solution for the flat spin model of Tail Configuration Four are presented below in Table 1 along with the graphical (equilibrium spin technique) results of Ref. 15. These results indicate that the equilibrium spin technique, which is based on a reduced order model, can accurately predict the 6-DOF model solution of the flat spin conditions. The trim results also indicate that the steady spin roll angle is very small (0.5 deg), as is assumed in the graphical solution.

Dynamic Analysis by Numerical Linearization

As discussed earlier, the use of reduced order spin models can decrease the complexity of the dynamic analyses, and provide more physical insight than can be achieved from the study of the full order problem. The present numerical linearization analysis utilized a 3-DOF model which assumes

Table 1 Iterative trim solution for flat spin mode of Tail Configuration Four

Analysis method	α, deg	λ	ϕ, deg
Equilibrium spin technique	72.0	0.60	$\equiv 0$
6 DOF iterative trim solution	71.2	0.57	0.5

Table 2 Comparison of the eigenvalue results for various dynamic modes and analysis techniques

	Rotary-oscillatory aerodynamic model		
Dynamic model	Mode A	Mode B	Mode C
3-DOF numerical linearization	$-0.16 \pm j6.11$	$-0.12 \pm j4.53$	$0.04 \pm j0.05$
3-DOF classical perturbation	$-0.13 \pm j6.14$	$-0.30 \pm j4.56$	$-0.09 \pm j0.11$
2-DOF (pitch-roll modes) classical perturbation	$-0.08 \pm j5.93$	$-0.35 \pm j4.57$	...
1-DOF (yawing mode) classical perturbation	...	...	$-0.09 \pm j0.11$
2-DOF double pole approx. (equal roots)	$-0.22 \pm j5.21$		...

	Pure rotary balance aerodynamic model		
Dynamic model	Mode A	Mode B	Mode C
3-DOF numerical linearization	$0.09 \pm j6.12$	$0.03 \pm j4.52$	$0.04 \pm j0.05$
3-DOF classical perturbation	$0.11 \pm j6.15$	$-0.14 \pm j4.56$	$-0.09 \pm j0.11$
2-DOF (pitch-roll modes) classical perturbation	$0.16 \pm j5.94$	$-0.19 \pm j4.57$	...
1-DOF (yawing mode) classical perturbation	...	...	$-0.09 \pm j0.11$
2-DOF double pole approx. (equal roots)	$-0.02 \pm j5.21$		...

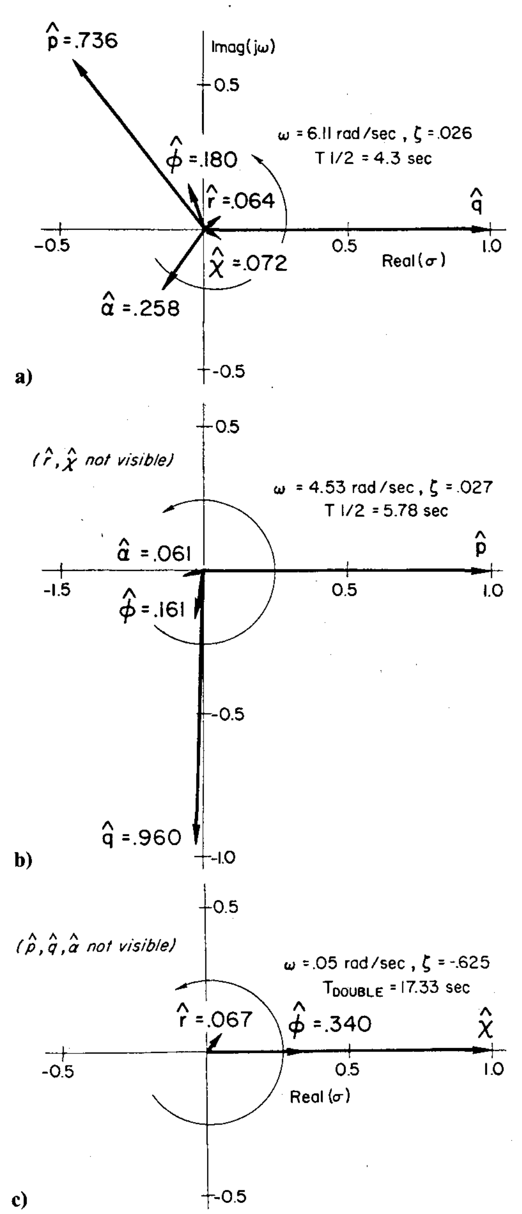

Fig. 2 Eigenvector results of the numerical linearization analysis of the 3-DOF model with a rotary-oscillatory data base. a) Mode A, b) mode B, c) mode C.

constant forces, such that the aircraft center of gravity will travel at a steady speed on a helical path, but allows arbitrary angular motion about the center of gravity. The values of spin rate Ω, spin radius R, and rate of descent $V_{z'}$, are all assumed constant, equal to their initial (steady spin) values. As a result, the standard nine essential state variables reduce to six variables for this model, namely, p, q, r, α, χ, ϕ. The governing equations[16] were numerically linearized about the flat spin attitude using the spline-fitted aerodynamic functions. The resulting eigenvalue-eigenvector modal (time vector) diagrams are presented in Fig. 2.

These numerically linearized results indicate the existence of three oscillatory dynamic modes, designated A, B, C. Modes A and B are marginally stable with damping ratios of 0.026 and 0.027 (time-to-half-amplitude, $T_{1/2}$, of 4.3 and 5.8 s), respectively, and oscillation frequencies of 6.11 and 4.53 rad/s, respectively, close to the spin frequency of 4.67 rad/s. Mode C has a much lower frequency of $\omega = 0.05$ rad/s and is marginally unstable, $\zeta = -0.625$ (time-to-double-amplitude, T_{double}, of 17.3 s).

The eigenvector results of Fig. 2 show both modes A and B to be coupled pitch-roll gyrations; $\hat{p}$, $\hat{q}$, $\hat{\alpha}$, and $\hat{\phi}$ being the primary components. The nearly equal relative magnitude of the $\hat{p}$ and $\hat{q}$ eigenvectors indicates that the amplitude of oscillation in pitch and roll is the same, the latter motion leading the former by a phase angle somewhat greater than 90 deg. The resulting motions are a sort of "coning" oscillation about the trim attitude. Mode C is a slow oscillatory motion in the variables r, χ, and ϕ; yaw and roll angle variations being in phase. This mode can be thought of as a slow, undamped yawing motion which is superimposed on the higher frequency coning motion of modes A and B.

An indication of the importance of the oscillatory derivatives on the stability of spinning motion was gained by repeating the numerical linearization analyses with these derivatives excluded. These eigenvalue results, using the pure rotary balance aerodynamic model, are presented in Table 2.

Modes A and B become marginally unstable, with $\zeta = -0.015$ and -0.007 (times-to-double-amplitude of 23.1 and 7.7 s), respectively. The frequency and eigenvectors were essentially unaffected by the omission of the oscillatory derivatives. Mode C, the yawing oscillation, was not at all affected because the yaw damping derivative $N_{r_{\text{osc}}}$ was omitted in both analyses.

These results indicate the importance of the dynamic derivatives in determining the stability characteristics of the spinning motion of general aviation configurations. The omission of $N_{r_{\text{osc}}}$ was felt to cause the instability of mode C.

The significant frequency separation of modes A and B from mode C suggests the formulation of a 2-DOF pitch-roll model to represent modes A and B, and a 1-DOF yawing model to represent mode C. These simplified models, and the stability analyses which employed them, are discussed in a later section.

Dynamic Analysis by Classical Perturbation Techniques

The governing equations of the 3-DOF model, analyzed numerically earlier, were linearized by the classical (analytical) perturbation techniques.[26] The aerodynamic data, which were stored as spline fits in the previous analyses, were converted to dimensional derivatives manually. The differences in the resulting aerodynamic derivatives, between the computer and manual techniques, were significant in some cases [notably $(L_p)_{RB}$, $(N_r)_{RB}$]. Therefore the eigenvalue-eigenvector results should not be expected to match precisely. The numerically generated derivatives are considered to constitute a more representative model.

The eigenvalues were obtained from the perturbationally linearized characteristic matrix with and without the oscillatory derivatives included. These results are presented for comparison with the previous numerically linearized results in Table 2. The frequency of oscillation (imaginary part) results of the modes for both cases compares favorably. Some discrepancies, most noticeably in modes B and C of the pure rotary balance model, occur in the damping (real part) results. These differences are felt to arise predominantly from the manually calculated aerodynamic derivatives, although some variations due to the differences in the equation linearization methods should be expected in any case. The eigenvector results from the perturbational equations compared very favorably with the results of Fig. 2.

The numerical linearization method has previously been shown to produce modal representations which accurately depict nonlinear 6 DOF time histories.[16] Therefore the overall good correlation of the present results indicates the ability of the classical method to closely determine the dynamic modes of spinning aircraft.

Lower Order Approximations

The modal results of the numerical dynamic analysis suggest, as noted, the formulation of a 2-DOF pitch-roll model and a 1-DOF yaw model. These simplified representations of spinning motion enable closed form analyses to be completed by classical perturbation techniques, thereby obtaining convenient analytical approximations to the stability criteria. The closed form analysis of higher order models is impractical because of the strongly coupled aerodynamics and vehicle dynamics which lead to complex solutions.

The 2-DOF pitch-roll model was formulated from the 3-DOF equations of motion by eliminating the variables r and χ. The 1-DOF model was used to describe the low frequency yawing mode (mode C), which Fig. 2 indicates to be oscillations in the state variables r, χ. The classically linearized 3-DOF results showed $\hat{\chi}$ and $\hat{\phi}$ eigenvectors to be in phase, and scaled according to the approximate equation $\hat{\phi}=0.32\,\hat{\chi}$. As such, the number of state variables was reduced to a 1-DOF set, namely, r, χ.

Table 3 Order-of-magnitude comparison of the constants a_0, a_1, and a_3

Constant	Rotary-oscillatory aerodynamic model	Rotary balance aerodynamic model
a_0	6×10^2	6×10^2
a_1	2×10^1	5×10^0
a_2	5×10^1	5×10^1
a_3	1×10^0	7×10^{-2}

The eigenvalues for the simplified and linearized equations of motion are presented in Table 2. The correlation of these results with the complete classical perturbation 3-DOF results is seen to be excellent for both aerodynamic models, thereby establishing the validity of the 2- and 1-DOF models of dynamic spinning motion. The characteristic matrices for these simplified models were evaluated in closed form to obtain analytical expressions for the characteristic polynomials.

Two-DOF Analytical Stability Analysis

The characteristic polynomial $p(s)$ for the 2-DOF model is of the form

$$p(s)=s^4+a_3s^3+a_2s^2+a_1s+a_0 \tag{2}$$

where s is the Laplace operator, and a_0, a_1, a_2, and a_3 are constant coefficients which are algebraic combinations of the rotary and oscillatory derivatives.

The necessary and sufficient conditions for stability are

a) positive coefficients:

$$a_0, a_1, a_2, a_3 > 0 \tag{3}$$

b) positive Routh discriminant:

$$a_3a_2a_1-a_1^2-a_3^2a_0>0 \tag{4}$$

Condition a, positive polynomial constants, was applied to the 2-DOF characteristic polynomial. Numerical values for the derivatives were substituted for the analytical expressions to make order of magnitude simplifications. A presentation of comparative orders of magnitude a_0, a_1, a_2, and a_3 is given in Table 3. Literal approximate expressions for these constants are given in Table 4.

The constants a_0, a_1, and a_2 are positive and large for zero and nonzero values of the oscillatory derivatives. The last term in Eq. (2), a_0, which determines the static stability conditions, is the largest of the constants and contains, as expected, no oscillatory derivatives. The condition on the smallest constant, a_3, comprises the important stability criterion a (for the present configuration). The expression for a_3 is given in Table 4, where $(L_p)_{RB}$ is the rotary balance roll damping term which arises from the resolution method for rotary data implementation.

This a_3 criterion provides some very useful information on the significance of oscillatory balance derivatives. For the present configuration, the first term, $-(L_p)_{RB}$, is positive but very small, contributing less than 10% of the total value of a_3. With the omission of the oscillatory derivatives, the positive value of a_3 drops, approaching zero. As a result, the roots of the characteristic Eq. (2) will be driven towards the imaginary axis—the stability-instability boundary. This criterion clearly explains the sensitivity of the eigenvalue results, presented in Table 2, to the omission of the oscillatory derivative. The possible significance of the a_1 constant, for other configurations, has previously been noted.[27]

Table 4 Approximate expressions for the 2-DOF model polynomial coefficients

Coefficient	Approximate literal expression
a_3	$-(L_p)_{RB}-L_{p_{osc}}-M_{q_{osc}}$
a_2	$-M_\alpha-[M_{p_I}+(M_p)_{RB}]L_{q_I}-L_\phi+r^2$
a_1	$-(M_\phi+M_{\phi_{osc}})(r+L_{q_I})$ $+(L_\alpha+L_{\alpha_{osc}})[r-M_{p_I}-(M_p)_{RB}]$ $+M_{q_{osc}}(L_\phi-r^2)+L_{p_{osc}}(M_\alpha-r^2)$
a_0	$-[M_{p_I}+(M_p)_{RB}]L_{q_I}r^2-(M_p+M_{p_I})L_\phi r$ $+L_{q_I}M_\alpha r+M_\alpha L_\phi$

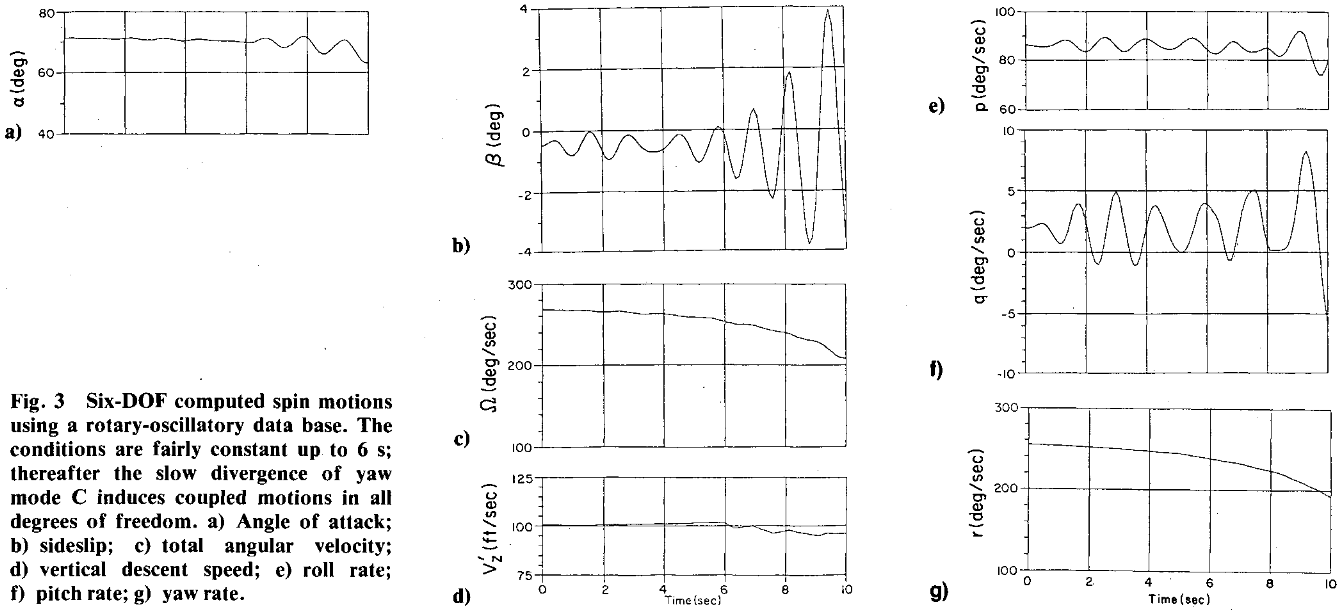

Fig. 3 Six-DOF computed spin motions using a rotary-oscillatory data base. The conditions are fairly constant up to 6 s; thereafter the slow divergence of yaw mode C induces coupled motions in all degrees of freedom. a) Angle of attack; b) sideslip; c) total angular velocity; d) vertical descent speed; e) roll rate; f) pitch rate; g) yaw rate.

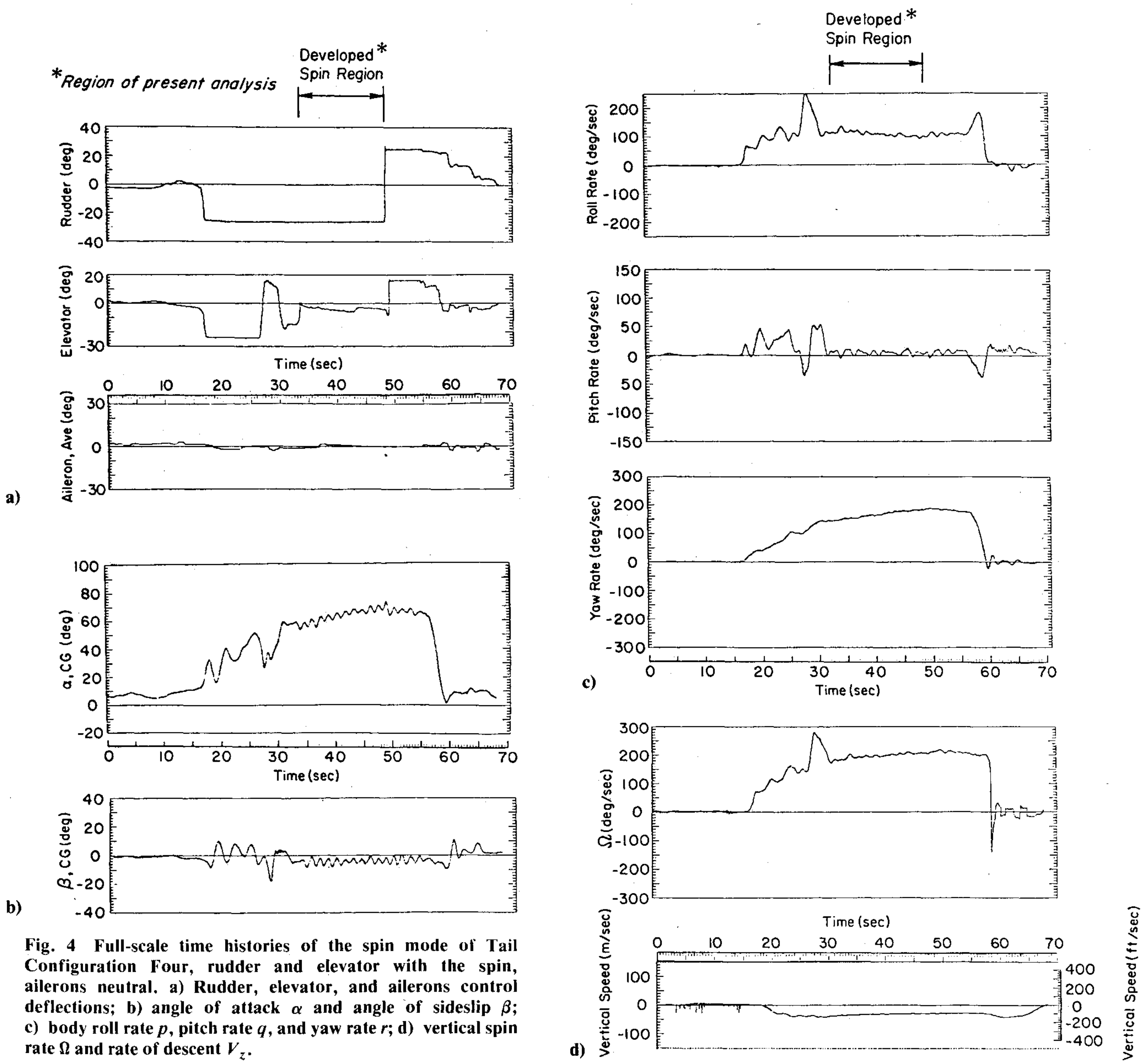

Fig. 4 Full-scale time histories of the spin mode of Tail Configuration Four, rudder and elevator with the spin, ailerons neutral. a) Rudder, elevator, and ailerons control deflections; b) angle of attack α and angle of sideslip β; c) body roll rate p, pitch rate q, and yaw rate r; d) vertical spin rate Ω and rate of descent V_z.

Condition b, Routh's discriminant, is a complicated combination of rotary balance and oscillatory derivatives. This criterion was evaluated numerically in the analysis for comparison with the eigenvalue results of Table 2. A value of 38.62 was calculated for the rotary-oscillatory aerodynamic model which, as expected from the previous results, indicates dynamic stability. The pure rotary balance aerodynamic model produced a value for Routh's discriminant of −7.11, indicating dynamic instability. This result is also in agreement with the eigenvalue solutions presented in Table 2.

Routh's discriminant, because of its complexity, is not a practical criterion for the present closed form analytical study. An alternative criterion for determining the dynamic stability of the pitch-roll modes (modes A and B) can be obtained by making the following "double pole" (equal roots) approximation for the characteristic polynomial of Eq. (2):

$$p(s) = [s-(\sigma+j\omega)]^2[s-(\sigma-j\omega)]^2 \tag{5}$$

For such a case, the eigenvalues for the preceding 2-DOF analysis may be calculated in closed form from the relations

$$\sigma \doteq -a_3/4 \tag{6}$$

and

$$\omega \doteq a_0^{1/4} \tag{7}$$

where a_3 and a_0 are given in Table 4; M_{p_I}, L_{q_I} are the "gyroscopic" pitching moment derivative with respect to roll rate, and rolling moment derivative with respect to pitch rate, respectively; and r is the steady-state (trim) yaw rate given approximately by $r \doteq \Omega_{\text{trim}}(\sin\alpha)$. Equation (6) shows the dynamic damping of the pitch-roll (2-DOF) modes to be a direct function of the oscillatory derivatives. The frequency, determined from Eq. (7), is controlled by rotary balance and gyroscopic derivatives.

The validity of Eqs. (5-7) was checked by the numerical application of these relations to the pure rotary and oscillatory-rotary data bases. The results are presented in Table 2. In both cases, the approximate roots obtained from Eqs. (5-7) were averages between the eigenvalues of modes A and B. As expected from Eq. (7), the frequency of oscillation remained unaffected by the omission of damping derivatives.

One-DOF Analytical Stability Analysis

The 1-DOF characteristic matrix was expanded analytically to produce the following exact results, numerically evaluated in Table 2:

$$\sigma = \frac{(N_r)_{\text{RB}}}{2} \tag{8}$$

$$\omega = \frac{[(N_r)^2_{\text{RB}} + 4(N_\chi + 0.32N_\phi)]^{1/2}}{2} \tag{9}$$

where $(N_r)_{\text{RB}}$ is the rotary balance yaw damping term which arises from the resolution method for rotary data implementation. The yaw mode stability criterion arising from Eq. (8) is

$$(N_r)_{\text{RB}} < 0 \tag{10}$$

The criterion is identical to that given by Graham for fighter configurations.[8] The presence of a pure yawing derivative criterion for a 1-DOF yawing model is not surprising. However, this result does suggest that an accurate value of $N_{r_{\text{osc}}}$, omitted throughout this study, could significantly affect the stability of this motion, thereby providing the necessary mode C damping that was lacking in these analyses. The inclusion of this derivative into the 1-DOF characteristic matrix results in the following criterion:

$$(N_r)_{\text{RB}} + N_{r_{\text{osc}}} < 0 \tag{11}$$

which resembles that given for a_3, the 2-DOF model stability criterion.

As demonstrated from the results of Table 2, the simplified eigenvalue solutions of Eqs. (5-9) provide useful quantitative information on the modal characteristics of the 2- and 1-DOF models. The application of these approximate equations provides an adequate assessment of the dynamic nature of the calculated steady flat spin mode without recourse to the previous, more complex computer-aided methods. Further experience with these criteria should assess their ability to predict the flat spin stability characteristics of other configurations, including those which may exhibit so-called "oscillatory spins." The sensitivity of the results to the sideslip derivatives, oscillatory derivatives, and manual derivative calculation method stresses the importance of obtaining more complete data bases, with finer increments in future wind-tunnel test programs.

Unconstrained 6-DOF Time History Results

Time histories for the rotary-oscillatory data base were generated in order to present the full nonlinear motion of the spinning aircraft for comparison with the extensive linearized analysis results of this study. These time histories are plotted in Fig. 3. The simulation results exhibit small bounded oscillations in the variables α, β, p, and q for the first 6 s. The yawing rate r and spin rate Ω are aperiodic and fairly constant for this interval. After this the instability of yaw mode C builds up and produces coupled motions in other degrees of freedom. The vertical descent velocity $V_{z'}$ is seen to remain fairly constant. Recall that steady c.g. motion was an initial assumption of all of the reduced order spin models used in this study.

The frequency of oscillation from the time histories was determined to be about 5.0 rad/s. Comparison with the value of 5.21 in Table 2 indicates the accuracy of the double-pole approximation. The stability of motion of all of the variables except sideslip correlates well with the earlier 3-DOF stable results. The instability in sideslip reflects the marginal instability of mode C indicated by the numerically linearized results. This instability would probably not have occurred if a more accurate value of $N_{r_{\text{osc}}}$ were incorporated in the oscillatory derivative model. Note, for example, at 4 s the oscillatory component of yaw rate ($r_{\text{osc}} \doteq r - \Omega\sin\alpha$) is negative; this, coupled with a negative (stable) value of $N_{r_{\text{osc}}}$, would propel the spin and thereby resist the declining yaw rate and resulting coupled instability.

The time histories are closely correlated with the modal characteristics of Fig. 2, exhibiting predominantly stable coupled pitch-roll motion with marginally stable sideslip variations generated by oscillations in χ and ϕ. These results validate the reduced order models and indicate usefulness of such simplified representations. Corresponding time histories for the full-scale aircraft[21] are presented in Fig. 4. A comparison of these results shows that the 6-DOF time histories, generated with a rotary-oscillatory data base, can realistically simulate actual flat spinning motion.

Time histories, utilizing only the rotary balance aerodynamic model, were not presented here. They exhibited immediate divergence in the variables α, β, p, and q—a result in agreement with the linearized results of Table 2.

Conclusions

1) The equilibrium spin states can be located accurately, using the graphical equilibrium spin technique, from an aerodynamic data base obtained from rotary balance testing. The previous graphical results were verified by unconstrained 6-DOF numerical solutions.

2) Three modes of motion about the flat spin attitude are identified, mode A: pitch-roll; mode B: pitch-roll; mode C: yaw.

3) For the flat spin, reduced order models provide good approximations to the 6-DOF calculations and allow stability criteria to be explicitly obtained in terms of rotary and oscillatory aerodynamic derivatives.

4) Oscillatory derivatives are required in order to model stable flat spinning motion.

5) Six-DOF time history computations with rotary balance data and estimated oscillatory derivatives closely match the flight data during the steady spin period.

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References

[1] Jones, B.M. and Trevelyan, A., "Step by Step Calculations Upon the Asymmetric Movements of Stalled Airplanes," ARC R&M 999, 1925.

[2] Bryant, L.W. and Jones, I.M.W., "Recovery from a Spin," ARC R&M 1428, 1932.

[3] Scher, S.H., "An Analytical Investigation of Airplane Spin-Recovery Motion by Use of Rotary-Balance Aerodynamic Data," NACA TN 3188, 1954.

[4] Anglin, E.L. and Scher, S.H., "Analytical Study of Aircraft Developed Spins and Determination of Moments Required for Satisfactory Spin Recovery," NASA TN D-2181, 1964.

[5] Anglin, E.L., "Recent Research on Aerodynamic Characteristics of Fighter Configurations During Spin," AIAA Paper 77-1163, 1977.

[6] Williams, D.H., "The Use of Rotary Balance Aerodynamics in Theoretical Spin Studies," M.S. Thesis, George Washington Univ., 1976.

[7] Kroll, W.B., "An Analytical Investigation of the Stall, Departure, and Spin Entry Characteristics of a Current Fighter Airplane Using Conventional and Rotary Aerodynamic Models," M.S. Thesis, George Washington Univ., 1976.

[8] Graham, A.B., "Prediction of Flat Spin Characteristics of a Fighter Airplane," M.S. Thesis, George Washington Univ., 1976.

[9] Gates, A.B. and Bryant, L.W., "The Spinning of Aeroplanes," ARC R&M 1001, 1926.

[10] Irving, H.B., "Simplified Presentation of the Subject of Spinning of Aeroplanes," ARC R&M 1426, 1932.

[11] Bazzocchi, E., "Etude Analytic de la Vrille en Utilisant des Donnees a la Soufflerie Horizontale et Method d' Essai des Models a l'Air Libre," *Proceedings of the Second European Aeronautical Congress*, Scheveningen, 1956.

[12] Simpson, E.J., "Macchi MB 326-H Spinning Characteristics Comparison of In-Flight Trials Data with Spinning Theory," Royal Australian Air Force, R&D Unit, Flight Test Report, 1973.

[13] Bazzocchi, E., "Stall Behavior and Spin Estimation Method by Use of Rotating Balance Measurement," *Stall/Spin Problems of Military Aircraft*, AGARD-CP-199, 1975.

[14] Tischler, M.B. and Barlow, J.B., "Application of the Equilibrium Spin Technique to a Typical Low-Wing General Aviation Design," AIAA Paper 79-1625, 1979.

[15] Tischler, M.B. and Barlow, J.B., "Determination of the Spin and Recovery Characteristics of a Typical Low-Wing General Aviation Design," *Journal of Aircraft*, Vol. 18, April 1981, pp. 238-244 (see also AIAA Paper 80-0169, Jan. 1980).

[16] Tischler, M.B., " Equilibrium Spin Analysis with an Application to a General Aviation Design," M.S. Thesis, Univ. of Maryland, 1979.

[17] Bihrle, W. Jr., Hultberg, R.S., and Mulcay, W.,"Rotary Balance Data for a Typical Single Engine Low-Wing General Aviation Design for an Angle-of-Attack Range of 30° to 90°," NASA CR-2977, 1978.

[18] Burk, S.M. Jr., Bowman, J.S. Jr., and White, W.L., "Spin-Tunnel Investigation of the Spinning Characteristics of Typical Single-Engine General Aviation Airplane Designs," NASA TP-1009, 1977.

[19] Full-scale flight test data of the NASA Low-Wing General Aviation Aircraft with Tail Configuration Three, courtesy of the NASA Langley Research Center, Hampton, Va, 1979.

[20] Hultberg, R.S. and Mulcay, W., "Rotary Balance Data for a Typical Single Engine General Aviation Design for an Angle of Attack Range of 8 deg to 90 deg, Vol. I: Low-Wing Model A," NASA CR-3100, Feb. 1980.

[21] Full-scale flight test data of the NASA Low-Wing General Aviation Aircraft with Tail Configuration Four, courtesy of the NASA Langley Research Center, Hampton, Va, 1979.

[22] Houck, J.A., Gibson, L.H., and Steinmetz, G.G.," A Real-Time Digital Computer Program for the Simulation of a Single-Rotor Helicopter," NASA TM X-2872, 1974.

[23] Burk, S.M. Jr., "Analytical Determination of the Mechanism of an Airplane Spin Recovery with Different Applied Yawing Moments by Use of Rotary-Balance Data," NACA 3321, 1954.

[24] Hoerner, S.F., *Fluid Dynamic Lift*, L.A. Hoerner, Brick Town, N.J., 1975.

[25] Bihrle, W. Jr., "Influence of the Static and Dynamic Aerodynamic Characteristics on the Spinning Motion of Aircraft," *Journal of Aircraft*, Vol. 8, Oct. 1971, pp. 764-768.

[26] Etkin, B., *Dynamics of Atmospheric Flight*, Wiley & Sons, New York, 1972.

[27] Tischler, M. B. and Barlow, J. B., "A Dynamic Analysis of the Motion of a Low-Wing General Aviation Aircraft About its Calculated Equilibrium Flat Spin Mode," *Proceedings of the AIAA 7th Atmospheric Flight Mechanics Conference*, Danvers, Mass., Aug. 1980, pp. 83-95.